Fluctuation Probes of Quark Deconfinement

Masayuki Asakawa[1], Ulrich Heinz[2], and Berndt Müller[3]
[1]Department of Physics, Nagoya University, Nagoya 464-8602, Japan
[2]Theoretical Physics Division, CERN, CH-1211 Geneva 23, Switzerland
[3]Department of Physics, Duke University, Durham, NC 27708-0305
(final version, February 1, 2008)

The size of the average fluctuations of net baryon number and electric charge in a finite volume of hadronic matter differs widely between the confined and deconfined phases. These differences may be exploited as indicators of the formation of a quark-gluon plasma in relativistic heavy-ion collisions, because fluctuations created in the initial state survive until freeze-out due to the rapid expansion of the hot fireball.

Fluctuations in the multiplicities and momentum distributions of particles emitted in relativistic heavy-ion collisions have been widely considered as probes of thermalization and the statistical nature of particle production in such reactions [1–6]. The characteristic behavior of temperature and pion multiplicity fluctuations in the final state has been proposed as a tool for the measurement of the specific heat [7] and, specifically, for the detection of a critical point in the nuclear matter phase diagram [8]. Although the hot and dense matter created in heavy-ion collisions is not directly observed at the critical point (if one exists) but rather at the point of thermal freeze-out where particles decouple from the system, certain features of the critical fluctuations were shown to survive due to the finite cooling rate of the fireball [9].

We here draw attention to a different type of fluctuations which are sensitive to the microscopic structure of the dense matter. If the expansion is too fast for local fluctuations to follow the mean thermodynamic evolution of the system, it makes sense to consider fluctuations of locally conserved quantities that show a distinctly different behavior in a hadron gas (HG) and a quark-gluon plasma (QGP). Characteristic features of the plasma phase may then survive in the finally observed fluctuations. This is most likely if subvolumes are considered which recede rapidly from each other due to a strong differential collective flow pattern as it is known to exist in the final stages of a relativistic heavy-ion reaction.

Three observables satisfy these constraints and are, in principle, measurable: the net baryon number, the net electric charge, and the net strangeness. Here we will focus on the first two as probes of the transition from hadronic matter to a deconfined QGP. Because they are sensitive to the microscopic structure of the matter, their unusual behavior would provide specific information about the structural change occurring as quarks are liberated and chiral symmetry is restored at high temperature. Our proposal differs from recent suggestions involving fluctuations in the abundance ratios of charged particles [10] and in the baryon number multiplicity [11] in that we only consider locally conserved quantities. We also disregard dynamical fluctuations of the baryon density caused by supercooling and bubble formation [12].

We consider matter which is meson-dominated, i.e. whose baryonic chemical potential μ and temperature T satisfy $\mu \lesssim T$. Our arguments will thus apply to heavy-ion collisions at CERN SPS energies and above. In the following, we first explain qualitatively how hadronic and quark matter differ with respect to net baryon number and electric charge fluctuations. We then present analytical calculations supporting the argument. Finally, we estimate the rate at which initial state fluctuations are washed out during the expansion of the hot matter in the final, hadronic stage before thermal freeze-out.

In a hadron gas nearly two thirds of the hadrons (for $\mu \ll T$ mostly pions) carry electric charge ± 1. In the deconfined QGP phase the charged quarks and antiquarks make up only about half the degrees of freedom, with charges of only $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$. Consequently, the fluctuation of one charged particle in or out of the considered subvolume produces a larger mean square fluctuation of the net electric charge if the system is in the HG phase. For baryon number fluctuations the situation is less obvious because in the HG baryon charge is now only carried by the heavy and less abundant baryons and antibaryons. Still, all of them carry unit baryon charge ± 1 while the quarks and antiquarks in the QGP only have baryon number $\pm\frac{1}{3}$. It turns out that, as $\mu/T \to 0$, the fluctuations are again larger in the HG, albeit by a smaller margin than for charge fluctuations. At SPS energies and below the difference between the two phases increases since the stopped net baryons from the incoming nuclei contribute to the fluctuations, and more so in the HG than in the QGP phase.

Generally, if $\mathcal{O}$ is conserved and μ is the associated chemical potential, in thermal equilibrium the mean square deviation of $\mathcal{O}$ is given by

$$(\Delta\mathcal{O})^2 \equiv \langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2 = T\,\frac{\partial\langle\mathcal{O}\rangle}{\partial\mu}\,, \tag{1}$$

where $\langle\mathcal{O}\rangle = \mathrm{Tr}\,\mathcal{O}\,e^{-(\mathcal{H}-\mu\mathcal{O})/T}\big/\mathrm{Tr}\,e^{-(\mathcal{H}-\mu\mathcal{O})/T}$. For $\mathcal{O} = N_b$ the r.h.s. of (1) is T times the *baryon number susceptibility* which was discussed earlier in the context of

possible signatures for chiral symmetry restoration in the hadron-quark transition [13].

In general, the relative fluctuation of any extensive variable vanishes in the thermodynamic limit $V \to \infty$ because the expectation value $\langle \mathcal{O} \rangle$ increases linearly with the volume V while the fluctuation $\Delta\mathcal{O}$ grows only like $\sqrt{V}$. In reality, the value of a conserved quantum number of an isolated system does not fluctuate at all. However, if we consider a small part of the system, which is large enough to neglect quantum fluctuations, but small enough that the entire system can be treated as a heat bath, Eq. (1) can be used to calculate the statistical uncertainty of the value of the observable in the subsystem. This is the scenario considered here.

We first discuss the fluctuations of the net baryon number. Since baryons are heavy, in the dilute HG phase we can apply the Boltzmann approximation [14]:

$$N_b^{\pm}(T,\mu) = N_b^{\pm}(T,0)\, \exp(\pm\mu/T)\,. \tag{2}$$

Here $N_b^{\pm}$ denotes the number of baryons (+) and antibaryons (−), respectively. The net baryon number is $N_b = N_b^+ - N_b^-$. Then the net baryon number fluctuations in the hadronic gas are given by

$$(\Delta N_b)^2_{\rm HG} = N_b^+ + N_b^- = 2\, N_b^{\pm}(T,0)\, \cosh(\mu/T)\,. \tag{3}$$

This result makes sense, because the fluctuation of either a baryon or an antibaryon into or out of the subvolume changes the net baryon number contained in it.

To estimate $(\Delta N_b)^2$ in the QGP phase, we use the exact result for the baryon number density in an ideal gas of massless quarks and gluons (for two massless flavors):

$$\frac{1}{V}\,(\Delta N_b)^2_{\rm QGP} = \frac{2}{9} T^3 \left(1 + \frac{1}{3}\Big(\frac{\mu}{\pi T}\Big)^2\right)\,, \tag{4}$$

where V denotes the volume of the considered subsystem. It is convenient to normalize this by the entropy density (again for two quark flavors plus gluons):

$$\frac{1}{V}\, S_{\rm QGP} = \frac{74\pi^2}{45}\, T^3 \left(1 + \frac{5}{37}\Big(\frac{\mu}{\pi T}\Big)^2\right)\,. \tag{5}$$

The later expansion being nearly isentropic, the ratio

$$\frac{(\Delta N_b)^2}{S}\bigg|_{\rm QGP} = \frac{5}{37\pi^2} \left(1 + \frac{22}{111}\Big(\frac{\mu}{\pi T}\Big)^2 + \dots\right)\,, \tag{6}$$

provides a useful measure for the fluctuations predicted for a transient quark phase. The entropy can be estimated from the final hadron multiplicity [16].

For high collision energies ($\mu/T \to 0$), the ratio (6) approaches a constant; even for SPS energies, the μ-dependent correction is at most 5%. The many resonance contributions make it difficult to write down an analytic expression like (5) for the entropy density in a hadron gas, but it is clear that the stronger μ-dependence of (3) compared to (4) induces a stronger μ-dependence of the corresponding ratio (6) in the HG phase. This translates into a stronger beam energy dependence of the ratio (6) near midrapidity in the HG than in the QGP phase.

Before providing numerical illustrations, let us compare these results with those for net charge fluctuations. All stable charged hadrons have unit electric charge; again using the Boltzmann approximation, which only for pions introduces a small error of at most 10%, we find

$$(\Delta Q)^2_{\rm HG} = N_{\rm ch}, \tag{7}$$

where $N_{\rm ch}$ is the total number of charged particles emitted from the subvolume. To find the expression for a noninteracting QGP, we introduce the electrochemical potential ϕ which couples to the electric charges $q_u = \frac{2}{3}$ and $q_d = -\frac{1}{3}$ of the up- and down-quarks:

$$\frac{\langle Q(\phi)\rangle}{V} = \sum_{f=u,d} q_f \left((\tfrac{1}{3}\mu + q_f\phi) T^2 + \frac{1}{\pi^2}(\tfrac{1}{3}\mu + q_f\phi)^3 \right). \tag{8}$$

We differentiate with respect to ϕ at $\phi = 0$ and normalize to the entropy density:

$$\frac{(\Delta Q)^2}{S}\bigg|_{\rm QGP} = \frac{25}{74\pi^2} \left(1 + \frac{22}{111}\Big(\frac{\mu}{\pi T}\Big)^2 + \dots\right). \tag{9}$$

This is a factor $\frac{5}{2}$ larger than the corresponding ratio (6) for baryon number fluctuations, due to the larger electric charge of the up-quarks, but shows the same weak μ-dependence. The main difference to baryon number fluctuations arises in the HG phase: Since at SPS and higher energies the r.h.s. of (7) is dominated by pions and meson resonances, its μ-dependence is now also weak. In contrast to baryon number fluctuations, charge fluctuations thus show a weak beam energy dependence in either phase, and only their absolute values differ [17].

We now give some numerical values for the fluctuation/entropy ratios at SPS and RHIC/LHC. At the SPS, the net baryon number per unit of rapidity is measured: $dN_b/dy \approx 92$ [18]. The antibaryon/baryon ratio is ≈ 0.085 [15,19], corresponding to $dN_b^-/dy \approx 8.5$. Combined with a specific entropy of $S/N_b \approx 36$ [20], Eq. (3) thus gives $(\Delta N_b)^2/S \approx 0.033$ if the fluctuations reflect an equilibrium HG. If they have a QGP origin, Eq. (6) gives $(\Delta N_b)^2/S \approx 0.014$ [14], i.e. about a factor 2.4 less. — The charge fluctuations in a HG can be evaluated from the measured charged multiplicity density at midrapidity, $dN_{\rm ch}/dy \approx 400$ [18,19,21], after correcting for resonance decays [16]. Assuming hadrochemical freeze-out at $T \approx 170\,\mathrm{MeV}$ [15], 60% of the observed pions stem from such decays [22]. One finds $(\Delta Q)^2/S \approx 0.06$. If the charge fluctuations arise from a QGP, Eq. (9) gives $(\Delta Q)^2/S \approx 0.036$, i.e. 60% of the HG value.

It is instructive to extrapolate these results to RHIC/LHC energies (i.e. $\mu/T \to 0$). We again assume hadrochemical freeze-out at $T \approx 170\,\mathrm{MeV}$ and use the particle

multiplicities predicted by hadrochemical models [23,24]. One obtains $(\Delta N_b)^2/S \approx 0.020$ in the HG, compared to 0.0137 in the QGP, and $(\Delta Q)^2/S \approx 0.067$ in the HG phase, compared to 0.034 in the QGP. Only the first of these four numbers, corresponding to the hadronic baryon number fluctuations, changes by more than 10% as one proceeds from SPS to RHIC (see Fig. 1).

These estimates, including our corrections for resonance decays, refer to ideal gases in equilibrium. Future work should address interaction effects on the thermal fluctuations in HG and QGP and treat resonance decays kinetically. We also point out potentially important non-equilibrium aspects: The fluctuation/entropy ratios in the QGP will be even lower (facilitating the discrimination against HG) if initially the QGP is strongly gluon-dominated [25] and hadronizes before the concentrations of the (baryon) charge carriers $q, \bar{q}$ saturate [26], or if hadronization itself generates additional entropy.

We now discuss whether the difference between the two phases (typically a factor 2) is really observable. Even if a QGP is temporarily created in a heavy-ion collision, all hadrons are emitted after re-hadronization. Thus, it is natural to ask whether the fluctuations will not always reflect the hadronic nature of the emitting environment. We must show that the time scale for the dissipation of an initial state fluctuation is larger than the duration from hadronization to final particle freeze-out. It is essential to our argument that fluctuations of conserved quantum numbers can only be changed by particle transport and thus are likely to be frozen in at an early stage, similar to the abundances of strange hadrons, which are frozen early in the reaction and may even reflect the chemical composition of a deconfined plasma [27].

For our estimate we assume for simplicity that the fireball expands mostly longitudinally, with a boost-invarant (Bjorken) flow profile. Longitudinal position and rapidity are then directly related. Strong longitudinal flow exists in collisions at the SPS [28], and the Bjorken picture is widely expected to hold for collisions at RHIC and LHC. Consider a slice of matter spanning a rapidity interval $\Delta\eta$ at the initial time τ_i. (τ is the proper time and $\eta = \tanh^{-1}(z/t)$.) Its proper volume is $V_i = A\tau_i\Delta\eta$ where A is the transverse area of the fireball. We denote the initial total baryon density by $\rho_i = \rho_{b+\bar{b}}(\tau_i)$. We assume $T_i = 170\,\mathrm{MeV}$ and $T_f = 120\,\mathrm{MeV}$ for the initial and final temperature [29], corresponding to $\tau_i \approx 2.5\,\mathrm{fm}/c$ and $\tau_f \approx 7\,\mathrm{fm}/c$ at the SPS, and $\tau_i \approx 5\,\mathrm{fm}/c$ and $\tau_f \approx 14\,\mathrm{fm}/c$ at RHIC.

Let us first give a qualitative argument [30] for the survival of a baryon number fluctuation within a rapidity interval $\Delta\eta \approx 1$. Between τ_i and τ_f, this interval expands from a length of 5 fm to 14 fm (we use the RHIC numbers here). Baryons have average thermal longitudinal velocity component $\bar{v}_z = \frac{1}{2}\bar{v}$ where $\bar{v} \equiv \langle|\boldsymbol{v}|\rangle = \sqrt{8T/\pi M}$ is the mean thermal velocity ($\bar{v} = 0.65$ for baryons with $M = 1\,\mathrm{GeV}$ at $T = 170$ MeV). Without rescattering, between τ_i and τ_f a baryon which is initially at the center of this interval can travel on average only about 3 fm in the beam direction; hence it will not reach the edge of the interval before freeze-out. Because of rescattering in the hot hadronic matter, the baryon number actually diffuses more slowly, and a fluctuation will even survive in a smaller rapidity interval.

For a quantitative argument, we need to estimate the flux of baryons in and out of the considered rapidity interval. Two effects need to be evaluated in this context. First, the difference in the baryon densities inside and outside the subvolume causes a difference in the values of the mean flux of baryons into and out of the volume. Denoting by $\bar{v}(\tau)$ the average thermal velocity of baryons, one finds that the initial fluctuation decays exponentially:

$$\Delta N_b(\tau) = \Delta N_b^{(i)} \exp\left(-\frac{1}{2\Delta\eta}\int_{\tau_i}^{\tau}\frac{d\tau}{\tau}\,\bar{v}(\tau)\right) . \qquad (10)$$

In the Bjorken scenario, the temperature T falls as $\tau^{-1/3}$ and one finds for the remaining fluctuation at freeze-out

$$\Delta N_b(\tau_f) = \Delta N_b^{(i)} \exp\left(-\frac{3\bar{v}_i}{\Delta\eta}\left[1-(T_f/T_i)^{1/2}\right]\right) . \qquad (11)$$

For the numbers considered here, the exponent is very close to $-\bar{v}_i/(2\Delta\eta)$, implying that the fluctuation survives if $\Delta\eta$ is larger than $\bar{v}_i/2 \approx 0.33$.

The second effect that can wash out the initial fluctuation is fluctuations in the baryon fluxes exchanged with the neighboring subvolumes. These could eventually replace the initial fluctuation with a thermal fluctuation that is characteristic of the conditions at freeze-out. The total number of baryons entering $N_b^{(\mathrm{en})}$ or leaving $N_b^{(\mathrm{lv})}$ the subvolume between τ_i and τ_f is given by

$$N_b^{(\mathrm{en})} = N_b^{(\mathrm{lv})} = \frac{A}{2}\int_{\tau_i}^{\tau_f}\rho_b(\tau)\,\bar{v}(\tau)\,d\tau\,. \qquad (12)$$

A similar calculation yields $N_b^{(\mathrm{en})} = N_b^{(\mathrm{lv})} \approx N_b^{(i)}\bar{v}_i/2\Delta\eta$. $N_b^{(\mathrm{en})}$ and $N_b^{(\mathrm{lv})}$ fluctuate independently; one therefore expects that the ratio of the mean square fluctuation of the number of exchanged baryons $N_b^{(\mathrm{ex})}$ to the average initial fluctuation is:

$$\frac{(\Delta N_b^{(\mathrm{ex})})^2}{(\Delta N_b^{(i)})^2} \approx \frac{\bar{v}_i}{\Delta\eta}\,, \qquad (13)$$

which is smaller than unity for $\Delta\eta \geq \bar{v}_i \approx 0.65$.

We conclude that the short time between hadronization and final freeze-out precludes the readjustment of net baryon number fluctuations in rapidity bins $\Delta\eta \geq 1$. A similar calculation applies to net charge fluctuations. Several refinements of our estimate are possible but are expected to partially cancel each other: Additional transverse expansion lets the temperature drop faster than in the Bjorken scenario. During hadronization cooling is

impeded by the large change in the entropy density between QGP and HG. And finally, the short mean free path of baryons in hot hadronic matter will significantly reduce our above estimates of the dissipation of an initial state fluctuation.

In conclusion, we have argued that the difference in magnitude of local fluctuations of the net baryon number and net electric charge between confined and deconfined hadronic matter is partially frozen at an early stage in relativistic heavy-ion collisions. These fluctuations may thus be useful probes of the temporary formation of a deconfined state in such collisions. The event-by-event fluctuations of the two suggested observables for collisions with a fixed value of the transverse energy dE_{T}/dy or of the energy measured in a zero-degree calorimeter would be appropriate observables that could test our predictions. Further discrimination can be achieved by measuring the beam energy dependence of the fluctuations: In the QGP the ratio $(\Delta Q/\Delta N_b)^2 = \frac{5}{2}$ of charge to baryon number fluctuations is a beam-energy independent constant; in the HG phase it shows a significant beam energy dependence between SPS and RHIC/LHC energies.

This work was supported by the U.S. Department of Energy, the Japanese Ministry of Education, Science, and Culture, and the Japanese Society for the Promotion of Science. B.M. thanks H. Minakata (Tokyo Metropolitan University) for his hospitality. We are indebted to K. Rajagopal for valuable discussions.

Note added: After finishing this work we received a paper by Jeon and Koch [31] who discuss similar issues. At the SPS they get $(\Delta Q)^2/S\big|_{\mathrm{HG}} \approx 0.13$ which is more than twice our value, due to a smaller resonance decay correction to $(\Delta Q)^2$ (30% instead of our 50%) and their omission of a 35% extra contribution to S [16] from heavy particles (mostly the net baryons and strange hadrons).

[1] M. Gaździcki and S. Mrówczyński, Z. Phys. C **54**, 127 (1992); S. Mrówczyński, Phys. Rev. C **57**, 1518 (1998); Phys. Lett. B **430**, 9 (1998); *ibid.* **459**, 13 (1999); *ibid.* **465**, 8 (1999); and nucl-th/9907099.

[2] E.V. Shuryak, Phys. Lett. B **423**, 9 (1998).

[3] A. Bialas and V. Koch, Phys. Lett. B **456**, 1 (1999).

[4] NA49 Collaboration, H. Appelshäuser *et al.*, Phys. Lett. B **459**, 679 (1999).

[5] G. Baym and H. Heiselberg, Phys. Lett. B **469**, 7 (1999).

[6] G. Danilov and E. Shuryak, nucl-th/9908027.

[7] L. Stodolsky, Phys. Rev. Lett. **75**, 1044 (1995).

[8] M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); Phys. Rev D **60**, 114028 (1999).

[9] B. Berdnikov and K. Rajagopal, Phys. Rev. D **61**, 105017 (2000).

[10] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999).

[11] S. Gavin and C. Pruneau, nucl-th/9906060; nucl-th/9907040.

[12] S. Gavin, nucl-th/9908070.

[13] L.D. McLerran, Phys. Rev. D **36**, 3291 (1987); S. Gottlieb *et al.*, Phys. Rev. Lett. **59**, 2247 (1987); Phys. Rev. D **38**, 2888 (1988); R.V. Gavai, J. Potvin, and S. Sanielevici, *ibid.* **40**, 2743 (1989).

[14] Our formulae ignore strange particles. This will do for a semiquantitative understanding. Strange particles carry an additional chemical potential μ_{S} to ensure strangeness conservation. It vanishes as $\mu \to 0$. In Pb-Pb collisions at the SPS, $\mu_{\mathrm{S}}/T \approx 0.42$ at hadrochemical freeze-out while $\mu/T \approx 1.58$ [15]. Our numerical examples below take this into account.

[15] P. Braun-Munzinger, I. Heppe, and J. Stachel, Phys. Lett. B **465**, 15 (1999).

[16] J. Sollfrank and U. Heinz, Phys. Lett. B **289**, 132 (1992).

[17] We emphasize that measuring $(\Delta N_b)^2$ requires the detection of (anti)neutrons. The net proton number is not conserved, and its fluctuations are dominated by isospin fluctuations induced by the fluctuating π^+/π^- ratio [10].

[18] NA49 Collaboration, G. Roland *et al.*, Nucl. Phys. **A638**, 91c (1998).

[19] NA49 Collaboration, P.G. Jones *et al.*, Nucl. Phys. **A610**, 188c (1996).

[20] J. Cleymans and K. Redlich, Phys. Rev. C **60**, 054908 (1999).

[21] NA49 Collaboration, F. Sikler *et al.*, Nucl. Phys. **A661**, 45c (1999).

[22] J. Sollfrank, P. Koch, and U. Heinz, Z. Phys. C **52**, 593 (1991).

[23] J. Zimányi, Nucl. Phys. **A661**, 224c (1999).

[24] J. Stachel, Nucl. Phys. **A661**, 226c (1999).

[25] E.V. Shuryak, Phys. Rev. Lett. **68**, 3270 (1992).

[26] T.S. Biro *et al.*, Phys. Rev. C **48**, 1275 (1993).

[27] A. Bialas, Phys. Lett. B **442**, 449 (1998); J. Zimányi, T.S. Biró, T. Csörgő, and P. Lévai, *ibid.* **472**, 243 (2000); J. Letessier and J. Rafelski, Phys. Rev. C **59**, 947 (1999).

[28] NA49 Collaboration, H. Appelshäuser *et al.*, Eur. Phys. J. **C2**, 661 (1998).

[29] R. Stock, Nucl. Phys. **A661**, 282c (1999).

[30] We thank K. Rajagopal for this observation.

[31] S. Jeon and V. Koch, hep-ph/0003168.

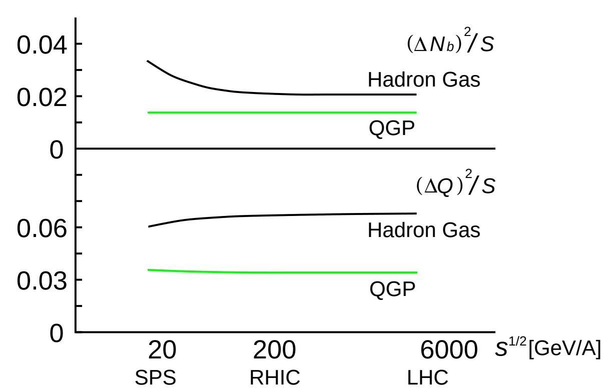

FIG. 1. Schematic drawing of the beam energy dependence of the net baryon number and charge fluctuations per unit entropy for a hadronic gas and a quark-gluon plasma.